Manipulation of Liquid Using Cases in Virtual Space

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Abstract

This paper describes a new method to realize an interactive manipulation of virtual liquid using a virtual case. The proposed model represents the liquid in the free fall condition as particles, while it does that in the stay condition as volume. The main purpose of this model is the realization of virtual liquid manipulation, while the generation of high quality computer graphics images or simulating the exact behavior of the liquid, are not the main. Our system with this proposed model makes it possible to catch the liquid using the virtual case, to hold the liquid in it, then to flow away the liquid with tilting it. Also the system realizes the manipulation to skim the liquid from another liquid case.

1 Introduction

In the field of virtual reality for the recent years, many researches such as [1], [2], [3] have been performed for the interactive manipulation in virtual space. In the conventional researches, the limitation is that users can manipulate only the solid objects under these systems. However, there are many situations that manipulate liquid or gas object, as in the cooking or in the amusement.

In the field of the hydrodynamics, research on the analysis of behavior of the liquid has been performed, while in the computer graphics technology, the scene image in a fog or that of water current have been represented by the particle based system [4]. However, these studies cost much calculation time and the direct application to the interactive manipulation of the virtual liquid is still difficult.

In this paper, a new method to realize an interactive manipulation of virtual liquid using a virtual case [5] is proposed. The proposed model represents the liquid in the free fall condition as particles, while it does that in the stay condition as volume. The main purpose of this model is the realization of the virtual liquid manipulation, while the generation of high quality computer graphics images or simulating the exact behavior of the liquid, are not the main. A virtual case is introduced to detect the interference with liquid. We consider only the interaction between the virtual case and the liquid not that between the general object and the liquid.

Our system with this proposed model makes it possible to catch the liquid using the virtual case, to hold the liquid in it, then to flow away the liquid with tilting it. Also the system realizes the manipulation to skim the liquid from another liquid case.

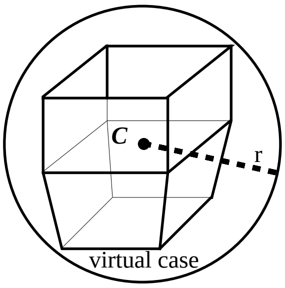

Figure 1: Virtual case and a sphere including it

2 Model for Virtual Liquid and Case

2.1 Virtual Liquid in Virtual Space

In this paper, we consider the liquid under the following conditions.

(1) free fall condition (such as flowing water from a cock).

(2) stay condition (such as holding water in a cup).

The proposed model represents the liquid in the condition (1) as particles. Each particle moves according to the gravity and the inertia. It is assumed that the size of the particle is ignored and each particle is not interfered with other particles.

While the liquid in the condition (2) is represented as volume. The exchange rate between each condition is assumed to be N [particles / volume]. When user falls all liquid by tilting the case of the volume V, the volume of liquid in it becomes 0 and the number of particles of liquid in free fall condition increases by NV.

2.2 Model of Virtual Case

We consider a case with the convex shape and a sphere including the case (Fig. 1). The center of the sphere is designated as C and its radius is designated as r. Then, the position of the case is represented by C. The direction of the case is represented by θ, ϕ and ψ (Fig. 2), while the tilt is represented by θ and ϕ. When θ and ϕ of a case are set to be 0 respectively, the status means that it is not tilted.

Fig. 3 shows a virtual case with the liquid. First, functions of the case with the liquid are defined in the following. The over flow point F is represented by the relative vector from the point C.

$$F = f(\theta, \phi, \psi). \tag{1}$$

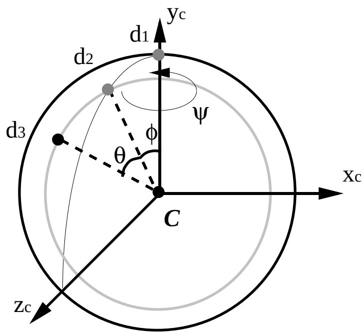

Figure 2: Representation of tilting of virtual case

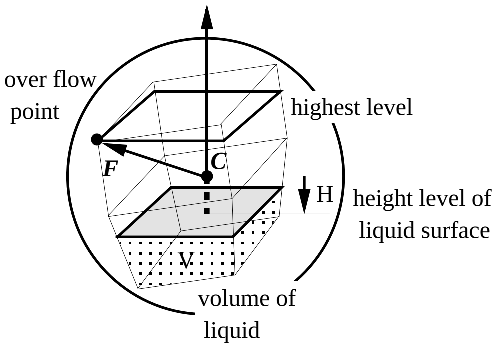

Figure 3: Each parameter of virtual case

When the volume of the liquid in a case is $V (V \geq 0)$, the height level of the liquid surface H is represented by the relative distance from the point C.

$$H = h(\theta, \phi, V). \tag{2}$$

When the parameter V of the function h is over the maximum volume Vm, H becomes F_y (F_y is defined as the vertical coordinate of F). The above equation is translated to

$$V = v(\theta, \phi, H). \tag{3}$$

When the parameter H of the function v is under the case, V becomes 0. When H is over the point F, V is taken as $v(\theta, \phi, F_y) = Vm(\theta, \phi)$ (F_y is independent of ψ).

When the height level is H, define the volume under H considering the thickness of case (Fig. 4) as

$$V^+ = v^+(\theta, \phi, H). \tag{4}$$

When the volume of liquid is Vm, V^+ becomes Vm^+. When the thickness of case is not considered, V^+ is regarded as V.

3 Interaction Model of Liquid and Case

First, the system gets the position and the direction $(C(t), \theta(t), \phi(t), \psi(t))$ of the case moved by the user in the virtual space. (This can be done using the 3-D position sensor.) In the following discussions, the value at time t is assumed when the description of the parameter t is omitted. Each particle is moved according to the gravity and the inertia. The particles moved under the predefined threshold plane will be deleted for the drawing.

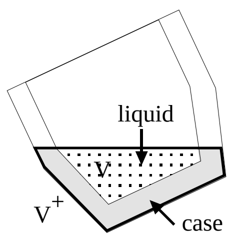

Figure 4: Volume considering thickness of virtual case

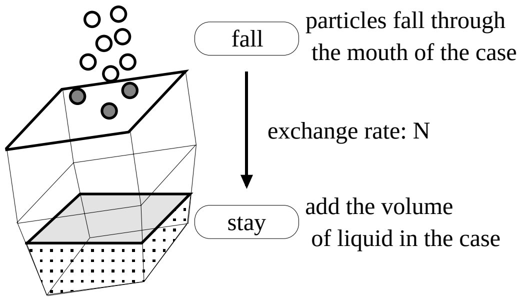

Figure 5: Relation for two conditions of liquid (1)

3.1 Liquid in Free Fall Condition and Case

When the falling liquid (each particle) moves through the mouth of a case, the status of the liquid changes from the condition (1) to the condition (2), and the volume of the liquid in it will be increased according to the exchange rate N (Fig. 5).

$$V(t)^{(0)} = V(t - \Delta t) + n/N, \tag{5}$$

where $n\,(n \geq 0)$ means the number of particles which move through the mouth and $V(t)^{(0)}$ means the assumed volume. When the case dose not interfere with the liquid in another case, let the assumed volume of the liquid in the case at time t be $V(t)^{(1)} = V(t)^{(0)}$. Then the height level is updated according to

$$H(t) = h(\theta, \phi, V(t)^{(1)}). \tag{6}$$

3.2 Liquid in Stay Condition and Case

In this section, we describe the interaction model between a case and the liquid in another case (Fig. 6). In the following, let each case be designated as *case 1* and *case 2* respectively, also let the symbol of each case be designated as subscript 1 and 2 for the distinction. When next equation holds at time t, case 1 and the liquid in case 2 are interfered each other.

$$C_{1y} - r_1 < C_{2y} + h_2(\theta_2, \phi_2, V_2^{(0)}), \tag{7}$$

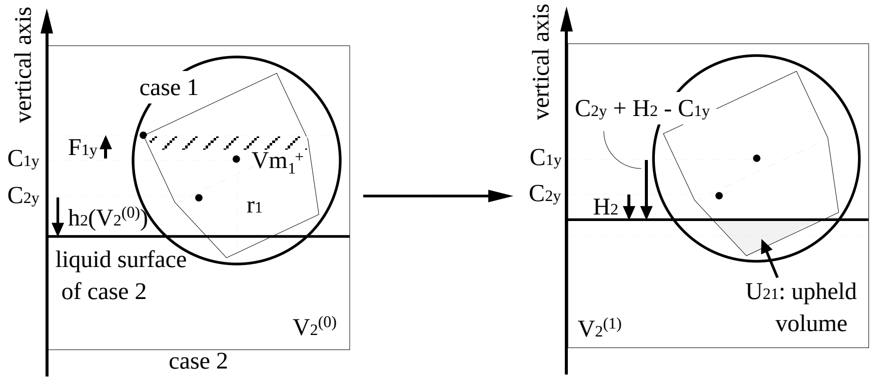

Figure 6: Interference between virtual case and liquid of another case

where C_{jy} is the vertical coordinate of the position C_j of case j. When the interference is detected, let the volume that case 1 upholds the height level of case 2 be $U_{21} \geq 0$. Let the volume that other cases uphold the height level of case 2 be

$$U_2 = \sum_j U_{2j}. \tag{8}$$

When case 1 and the liquid in case 2 are interfered each other and next equation holds, the situation is that the part of case 1 under the over flow point interferes with the liquid surface of case 2.

$$C_{2y} + h_2(\theta_2, \phi_2, V_2^{(0)} + Vm_1^+) < C_{1y} + F_{1y}. \tag{9}$$

When the above equation does not hold, the situation is that the liquid in case 2 flows into case 1.

3.2.1 Upholding of Liquid Surface

When a case and the liquid surface are interfered each other, case 1 upholds the liquid surface of case 2. The height level H_2 of case 2 should satisfy next equation.

$$\begin{aligned} H_2 &= h_2(\theta_2, \phi_2, V_2^{(0)} + U_{21}), \\ U_{21} &= v_1^+(\theta_1, \phi_1, C_{2y} + H_2 - C_{1y}). \end{aligned} \tag{10}$$

Let the volume in each case $V_j^{(0)}$ be updated as $V_j^{(1)}$. Then the height level of case 1 is determined from the equation (6).

3.2.2 Flowing of Liquid into Case

When the liquid in case 2 flows into case 1 and the next equation holds, the height level of liquid in case 2 should be the height of the over flow point of case 1 (Fig. 7).

$$\begin{aligned} V_2^{(0)} + U_{21} - v_2(\theta_2, \phi_2, C_{1y} + F_{1y} - C_{2y}) \\ < Vm_1 - V_1^{(0)}. \end{aligned} \tag{11}$$

When the liquid flows into a case, case 1 upholds the height level of case 2 with the volume $U_{21} = Vm_1^+$. The height level of case 2 should be the height of the over flow point of case 1 (equation (12)). The volume of each case is determined from the equations (13) and (14). Then, the height level of case 1 is determined from the equation (6).

$$H_2 = (C_{1y} + F_{1y}) - C_{2y}. \tag{12}$$

$$V_2^{(1)} = v_2(\theta_2, \phi_2, H_2) - U_{21}. \tag{13}$$

$$V_1^{(1)} + V_2^{(1)} = V_1^{(0)} + V_2^{(0)}. \tag{14}$$

When equation (11) does not hold, the liquid surface of case 2 should be higher than the height of the over flow point of case 1 (Fig. 8). Then, the volume $V_1^{(1)}$ and the height level H_1 of case 1 are set as the maximum volume Vm_1 and the radius r_1, respectively, where the liquid surface of case 1 is not considered (not drawn). The volume and the height level of case 2 are determined from the equation (14) and next equation.

$$H_2 = h_2(\theta_2, \phi_2, V_2^{(1)} + U_{21}). \tag{15}$$

When the liquid does not flow into another case, user can skim the liquid with the case by the above process.

3.3 Flowing Away Liquid from Case

When next equation holds at time t by the above interference and tilting of a case, the liquid flows away from the case (Fig. 9).

$$V^{(1)} + U > Vm. \tag{16}$$

The part of the liquid in the case, which is expressed using the volume, flows away at the over flow point F as $N(V^{(1)} + U - Vm)$ particles. The volume in the case at time t is determined as

$$V(t) = Vm - U. \tag{17}$$

While if the equation (16) does not hold, the liquid dose not flow away and the volume $V(t)$ becomes $V(t)^{(1)}$.

3.4 Drawing and Other Processes

The virtual cock which flows the infinite liquid is defined for the experiment of the interaction between a virtual case and the liquid in the free fall condition. After the above processes, liquid particles are generated from the cock.

Actual shape of the liquid surface of each case is generated for the drawing of the CG image based on the

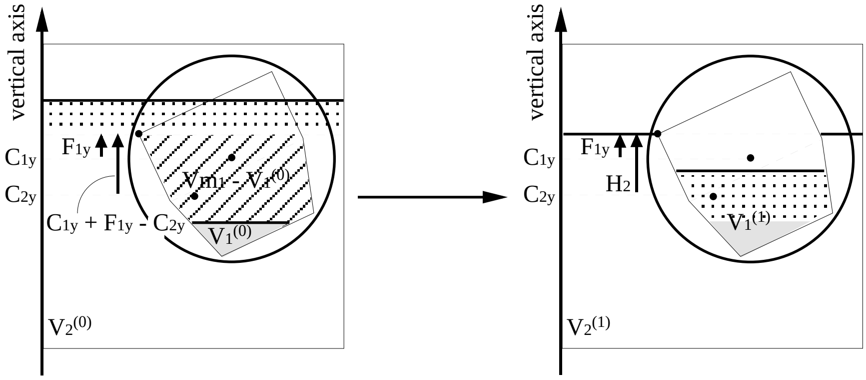

Figure 7: Flowing liquid into virtual case (1)

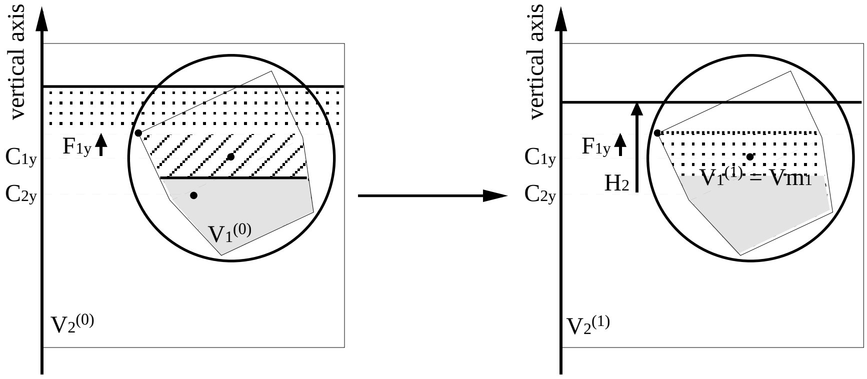

Figure 8: Flowing liquid into virtual case (2)

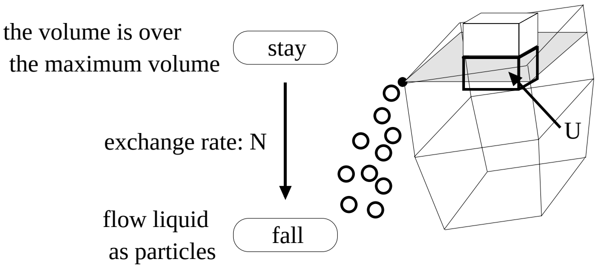

Figure 9: Relation for two conditions of liquid (2)

shape of the case and the height level. When case 1 interferes with the liquid in case 2 and the height level of case 2 is not higher than the over flow point of case 1, it is necessary that the part of the liquid surface of case 2 should be removed for case 1.

Then, each particle, each case, its liquid surface and its shadow are drawn.

4 System

For the implementation of a system, each function of each case expressed by equations (1) - (4) should be made based on the shape of each case respectively. In this experimental system, a hemisphere is used as a moved case 1 and a cube is used as a fixed case 2. Two volume functions with height level parameter are

$$v_1(H) = \pi(-H^3 + 3r_1^3H + 2r_1^3)/3, -r_1 < H < F_{1y}, \quad (18)$$

$$v_2(H) = a^2(H + a/2), -a/2 < H < a/2. \quad (19)$$

If the height level H is under the case, $v_j(H)$ is set to 0. If H is over F_{jy}, $v_j(H)$ is set to Vm_j. The thickness of the case is ignored in this system. Two height level functions with volume parameter can be obtained by translating the above functions. The over flow point of case 1, in general, can be obtained by the geometric method. In this paper, each geometrical expression is omitted to avoid the complexity. The over flow point of case 2 is set to one vertex and it is fixed.

Equation (10) to calculate the height level of case 2 that is upheld by case 1 can be solved by the algebraic method for these cases, i.e., a hemisphere and a cube. However, this system gets the height level by the following numerical method. First, the initial value of the height level of case 2 is set to the value that the liquid in case 2 is not interfered with case 1. Then, the height level is updated from the following equations until the

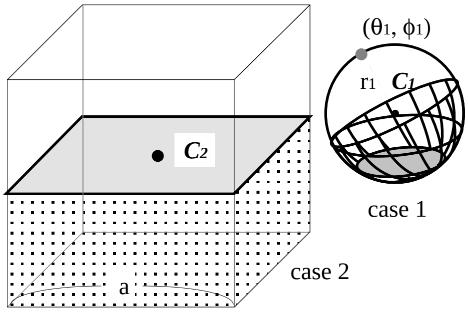

Figure 10: Shape of two cases defined in experiment system

height level of case 2 is converged.

$$
\begin{aligned}
H_{2(0)} &= h_2(\theta_2, \phi_2, V_2^{(0)}),\\
U_{21(i)} &= v_1^{+}(\theta_1, \phi_1, C_{2y} + H_{2(i)} - C_{1y}),\\
H_{2(i+1)} &= h_2(\theta_2, \phi_2, V_2^{(0)} + U_{21(i)}), i \geq 0. \quad (20)
\end{aligned}
$$

5 Experimental Results

Using the model mentioned above, we implemented a system for the manipulation of the virtual liquid using virtual cases in C language on a graphics workstation SGI O2. The exchange rate N is taken to be 10 [number of particles / cc]. The case can be moved by a user and it is implemented as a hemisphere whose radius is 5 cm, and another fixed case is a cube whose edge length is 30 cm (Fig. 10). In the experiment, 1 cc of the falling liquid is generated per 1 CG frame.

An example operation is shown in Fig. 11. This system gets the position and the direction of the real case via the motion sensor using a low-frequency magnetic field. As shown in the Fig. 12(left), the user can catch the falling liquid using the virtual case, hold the liquid in it, then flow away the liquid. Fig. 12(right) shows that the user can skim the liquid from another case. This system can refresh its screen at 24 frames/sec as the average speed for the representation that there are about 500 particles.

Through this virtual liquid manipulation system in our laboratory, we get some positive evaluations such that "I feel that I really manipulate the water (liquid) using a cup." The real cup with sensor is given to users and only what users can manipulate the water (liquid) in the screen using a real cup was explained to users. As the time to make users to feel that they can manipulate the liquid freely, it is 65, 75, 125, 41, 53, 42, 40, 48, 62 and 66 seconds respectively, and the average time is about 1 minute. From these results, it is considered that this system can keep the essential characteristics of real liquid manipulation. Other comments and the remained problems are as follows:

- They could recognize the feeling about the position of virtual cup from the shadow of it, but they tend to take notice to the shadow. They are able to catch the falling liquid more easily if the shadow of each particle is drawn.
- The feeling of the movement of a virtual cup is smaller than that of the actual movement of real cup. It is difficult to keep the virtual cup (the liquid surface in it) horizontally.

For the former, introducing stereo graphics may solve the problem of the sense of depth. For the latter, a calibration is necessary to consider the size and the position of the screen. Introducing the force feedback will also be required for the representation of weight of the liquid in a case.

6 Conclusion

In this paper, we proposed a model for the liquid manipulation using virtual cases and implemented an experimental system as a virtual reality system. Using the virtual case, the system can catch the falling liquid, skim the liquid from another case, hold the liquid and flow away the liquid at an interactive refresh rate. Users have an impression close to that in the real world.

The system proposed in this paper is under the stage of the realization of the basic functions, and the following points are remained as the future subjects.

- Manipulation of the liquid using the cases with the general shape which is expressed by polyhedron.
- Interaction with the liquid under the third condition such as the water current in the river.

The addition of these functions will be required. We consider the further realization method for a general polyhedron case. At starting up a system, it is considered that the LUT (look up table) is made to calculate the equations (1)-(4). It is also considered that a concave polyhedron case should be divided into some neighboring convex polyhedrons.

Acknowledgments

We thank our colleagues in our laboratory for useful discussions.

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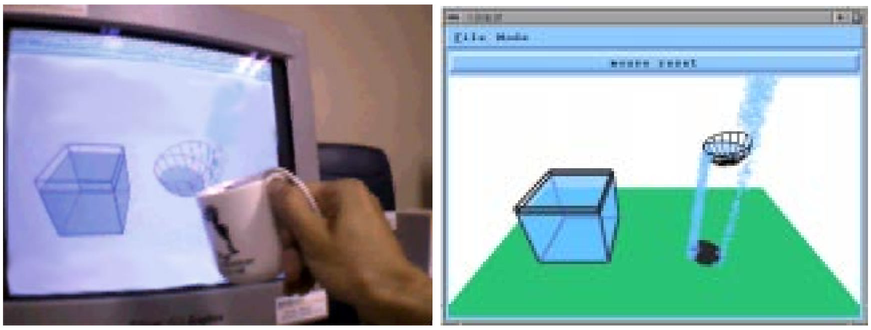

Figure 11: Appearance of proposed system

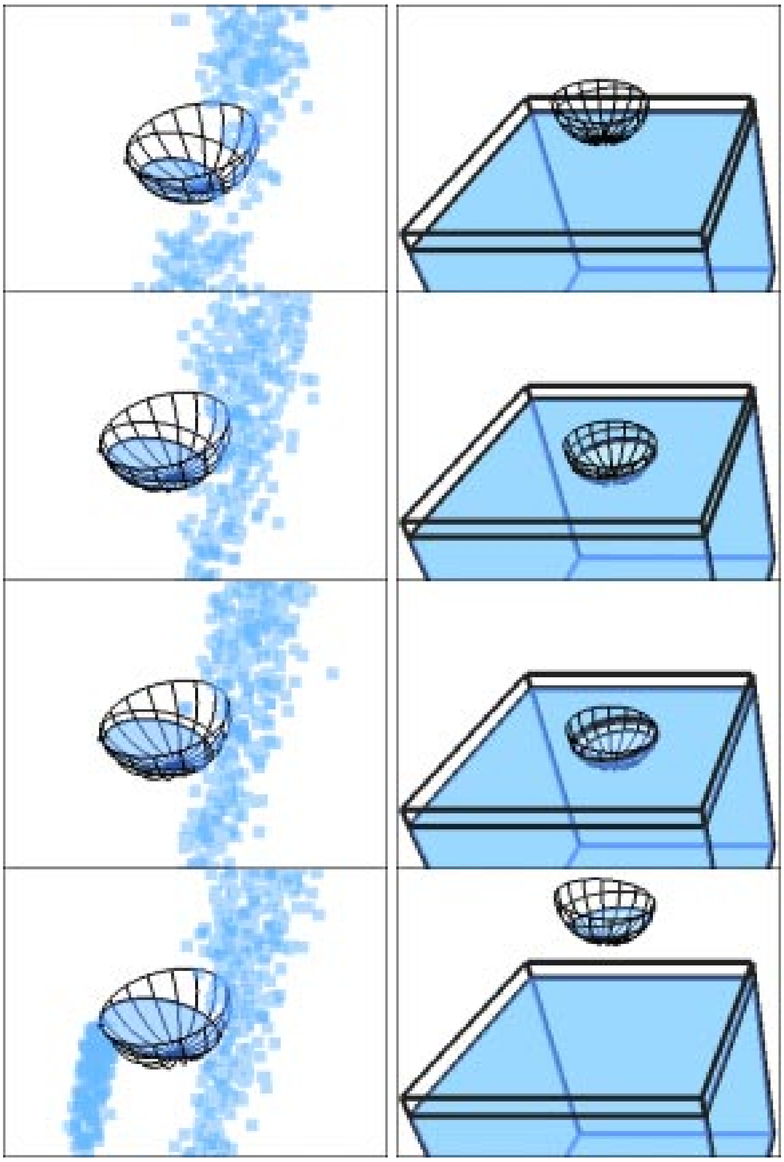

Figure 12: Examples of manipulation of liquid using virtual case